

EXTRA DIMENSIONS

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LECTURE 4

MIXING OF ZERO AND KK MODES FOR W, Z VIA HIGGS VEV

Zero and KK modes for $W_{i=1,2,3}$ and B (hypercharge)
defined with $v = 0$ (no kinetic/mass mixing)

$v \neq 0$: mass mixing for zero-modes (as in SM) \rightarrow

Define $Z_{\mu}^{(0)}, A_{\mu}^{(0)}$ as combinations of $W_3^{(0)}$ and $B^{(0)}$:

diagonalize 0-mode mass mixing (as in SM)

$$g_{W^{(0)}} = g_5/2/\sqrt{2\pi R + r}, \quad g_{Z^{(0)}} = g_5/2/\sqrt{2\pi R + r} \dots$$

$$(g_{5Z}^2 = g_{5/2}^2 + g_5'^2)$$

Define $Z^{(n)}$ and $A^{(n)}$ using same (0-mode) mixing angles

$A_{\mu}^{(n)}$ does not couple to Higgs (like zero-mode)

- Zero and KK modes of W mix via Higgs vev localized

at $y = \pi R$ (similarly for Z):

mass eigenstates are mixtures

diagonalize 2×2 mass matrix (zero and 1 KK mode)

for simplicity (homework 3)

2. SHIFT IN COUPLING TO Z

Shift in coupling of fermion at $y = 0$ from pure

zero-mode coupling,

due to (small) KK Z component of SM Z :

$$g_Z = g_{Z(0)} + \delta g_Z$$

Estimate via mass insertion diagrams

(valid for $v \times \text{couplings} \ll m_{KK}$):

$$\delta g_Z \sim g_{Z(0)}^2 v^2 / m_{KK}^2,$$

\rightarrow *no* enhancement in δg_Z for large brane kinetic terms ($r/R \gg 1$)

(enhancement at Higgs-KK Z vertex cancels suppression at fermion-KK Z vertex)

- Agrees with SM prediction at $\sim 0.1\%$ level \rightarrow

$$m_{KK} \gtrsim \text{a few TeV}$$

3. ρ PARAMETER

Shift in W mass from pure zero-mode mass (also for Z):

$$M_W^2 = M_{W(0)}^2 + \delta M_W^2 \quad (1)$$

$$M_{W(0)}^2 = \frac{1}{4} g_{W(0)}^2 v^2 \quad (2)$$

$$\frac{\delta M_W^2}{M_{W(0)}^2} \sim g_{W(0)}^2 \frac{v^2}{m_{KK}^2} \frac{r}{R} \quad (3)$$

What about

$$\rho = \frac{M_W^2}{M_Z^2} \times \frac{g_Z^2}{g_2^2} ? \quad (4)$$

$\rho = 1$ in SM (tree-level); $\Delta\rho_{expt.} = \rho_{expt.} - 1 \sim 10^{-3}$

Subtlety due to couplings modified from pure zero-mode:

$g_Z = g_{Z(0)} + \delta g_Z$, but δg_Z not enhanced by $r/R \gg 1$

\rightarrow set $g_Z \approx g_{Z(0)}$ in $\Delta\rho$

$$\delta\rho \equiv \rho - 1 \sim (g_{Z(0)}^2 - g_{W(0)}^2) \frac{v^2}{m_{KK}^2} \times \frac{r}{R} \quad (5)$$

- $\Delta\rho$ enhanced by large brane kinetic terms \rightarrow

$m_{KK} \gtrsim 10 \text{ TeV}$ for $r/R \sim 10$

CUSTODIAL ISOSPIN IN SM

Higgs potential, $V(|H|)$ with

$$H = (h_1, h_2, h_3, h_4) \quad (6)$$

has enhanced $SO(4) \approx SU(2)_L \times SU(2)_R$ symmetry
 $\rightarrow SO(3) \approx SU(2)_{\text{cust.}}$ by

$$\langle H \rangle = (0, 0, 0, v) \quad (7)$$

\rightarrow equal mass for $W_{i=1,2,3}^L$

W_3^L only mixes with B (no mixing for W_L^\pm)

$$\rightarrow M_Z^2 = 1/4 v^2 (g_2^2 + g'^2) \neq M_W^2 = 1/4 v^2 g_2^2$$

- Factor of g_Z^2/g_2^2 in definition $\rho = M_W^2/M_Z^2 g_Z^2/g_2^2$
takes this “violation of custodial symmetry” into
account

CUSTODIAL ISOSPIN VIOLATION IN $5D$

$\Delta\rho$ from KK $\propto (g_{Z^{(0)}}^2 - g_{W^{(0)}}^2 \sim g_{B^{(0)}}^2)$ as in SM

- *Additional* mixing (due to KK modes):

$W_{L3}^{(0)} - B^{(n)}$ (only in neutral sector)

no charged counterpart

$SU(2)_L \times U(1)_Y$ gauged in $5D \rightarrow$

KK's only for $W_L^{3\pm}$ and B

new effect not taken into account by factor of g_Z^2/g_2^2

in definition of ρ

$W_{L3}^{(0)} - B^{(n)}$ mass term

$$\sim g_{W^{(0)}} g_5' \times f_n(\pi R) v^2 \sim g_{W^{(0)}} g_{B^{(0)}} v^2 \sqrt{r/R}$$

\rightarrow enhanced for large brane terms!

$W_{L3}^{(0)} - W_{L3}^{(n)}$ mixing *does* have counterpart in charged sector

CUSTODIAL ISOSPIN SYMMETRY IN $5D$

Need extra KK modes to partner $B^{(n)}$:

promote to a triplet

- Restore custodial isospin by $SU(2)_L \times SU(2)_R$
gauged in $5D$ (hep-ph/0308036)

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ for fermion hypercharges:

$$Y = T_{3R} + (B - L)/2$$

$$T_{3R} = \pm 1/2 \text{ for } (u, d)_R \text{ and } (\nu, e)_R,$$

$$B - L = 1/3, -1 \text{ for } q, l \text{ (check)}$$

KK $U(1)_{B-L}$ do not couple to Higgs: only KK $W_{L,R}^{3\pm}$

KK exchanges respect custodial isospin

(same in charged and neutral channels)

BREAKING BY LARGE MASS ON BOUNDARY \equiv BOUNDARY CONDITION

Break $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$: no zero-modes for W_R^\pm and extra $U(1)$ (combination of $U(1)_R$ and $U(1)_{B-L}$ orthogonal to $U(1)_Y$)

- Breaking must approximately preserve degeneracy – of mass and coupling (to Higgs) – for (at least light)

W_R^\pm vs. W_R^3 modes to protect $\Delta\rho$

For large brane kinetic terms ($r/R \gg 1$),

KK's localized near $y = \pi R \rightarrow$

break on $y = 0$ brane, degeneracy not affected by breaking

Large mass term for W_R^\pm , extra $U(1)$ at $y = 0$ (from scalar vev) \equiv requiring vanishing at $y = 0$ (odd/Dirichlet boundary condition: section 3.3 of hep-ph/0404096)

SIGNALS (I)

- Coupling of KK gluon to top enhanced,
to light fermions suppressed

Real production of gauge KK

Broad resonance decaying into top pairs

challenge to distinguish from SM background:

use spin-correlation (spin-1 for KK gluon vs. not for
SM t -channel gluon exchange) or

dominantly decays to RH top (due to $Z \rightarrow \bar{b}b$)

- Distinguish from SUSY: no missing energy + top
special

SIGNALS (II)

Virtual exchange of gauge KK

1. $\bar{t}tZ$ shifted compared to $\bar{e}eZ$:
measure at ILC
2. Flavor violating coupling to KK $Z \rightarrow$
 $t \rightarrow cZ$ (at LHC)